Electro-weak $SU(4)_L \otimes U(1)_Y$ models without exotic electric charges

ADRIAN PALCU

Faculty of Exact Sciences - "Aurel Vlaicu" University Arad, Str. Elena Drăgoi 2, Arad - 310330, Romania

Abstract

For the particular class of $SU(4)_L \otimes U(1)_Y$ electro-weak models without exotic electric charges, some plausible phenomenological predictions - such as the boson mass spectrum and charges of all the fermions involved therein - are made by using the algebraical approach of the exactly solving method for gauge models with high symmetries. Along with the one-parameter resulting mass scale (to be confirmed at TeV scale in LHC, LEP, CDF and other high energy experiments) our approach predicts the exact expressions of the charges (both electric and neutral) in the fermion sector, while all the Standard Model phenomenology is naturally recovered.

PACS numbers: 12.10.Dm; 12.60.Fr; 12.60.Cn.

Key words: 3-4-1 gauge models, boson mass spectrum

1 Introduction

In view of new experimental challenges - such as tiny massive neutrinos and their oscillations or extra-neutral gauge bosons, to mention but a few - the Standard Model (SM) [1] - based on the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ that undergoes in its electro-weak sector a spontaneous symmetry breakdown (SSB) up to the electromagnetic universal one $U(1)_{em}$ - has to be properly extended. One of the most appealing extensions of the SM relies on the gauge symmetry $SU(3)_C \otimes SU(4)_L \otimes U(1)_Y$ (hereafter 3-4-1 model) that also undergoes a SSB up to $U(1)_{em}$. Its phenomenology was exploited in a series of recent papers [2] - [4] within the framework of the traditional approach. Apart from this, we treat here a particular class of such models (namely, the one without exotic electric charges) by resorting to the exact algebraical method proposed more than a decade ago by Cotǎescu [5]. This method, designed for gauge models with high symmetries with SSB, is based on a proper minimal Higgs mechanism (mHm) employing a promising parametrization in the scalar sector, and consequently setting the versors in the so called generalized Weinberg transformation (gWt) that separates the electromagnetic field and diagonalizes the mass matrix in the neutral (diagonal) bosons sector. As in the SM, only one neutral scalar field finally remains. Its vacuum expectation value (vev) $\langle \phi \rangle$ determines the overall breaking scale of the model.

We take for granted here the resulting formulas of the general method and apply them to the particular 3-4-1 model of interest here. For specific details of the method, the reader is referred to Ref [5]. We discriminate [6] among he various classes of such models by using the prescriptions suggested by the general method and we avoid the classification carried out in Ref. [4] which is based on the well-known parameters b and c. Notwithstanding, we recover almost the same classes of models, except for the one treated in Ref. [7].

Our paper is organized as follows. Sec. 2 briefly reviews the main features of the general method, while Sec. 3 presents the particle content of the 3-4-1 models investigated in this paper and computes both the mass terms in its boson sector and neutral charges of the fermions involved therein. A one-parametr mass scale is finally given for all the bosons. Sec. 3 presents our conclusions and phenomenological estimates.

2 The General Method

2.1 $SU(n)$ irreducible representations

The general method mainly relies on the two fundamental irreducible unitary representations (irreps) $\mathbf{n}$ and $\mathbf{n}^*$ of the $SU(n)$ group which are involved in contructing different classes of tensors of ranks (r,s) as direct products like $(\otimes \mathbf{n})^r \otimes (\otimes \mathbf{n}^*)^s$. These tensors have r lower and s upper indices for which we reserve the notation, $i, j, k, \cdots = 1, \cdots, n$. As usually, we denote the irrep ρ of $SU(n)$ by indicating its dimension, $\mathbf{n}_\rho$. The $su(n)$ algebra can be parameterized in different ways, but here it is convenient to use the hybrid basis of Ref. [5] consisting of $n-1$ diagonal generators of the Cartan subalgebra, $D_{\hat{i}}$, labeled by indices $\hat{i}, \hat{j}, ...$ ranging from 1 to $n-1$, and the generators $E^i_j = H^i_j / \sqrt{2}$, $i \neq j$, related to the off-diagonal real generators H^i_j [8]. This way the elements $\xi = D_{\hat{i}} \xi^{\hat{i}} + E^i_j \xi^j_i \in su(n)$ are now parameterized by $n-1$ real parameters, $\xi^{\hat{i}}$, and by $n(n-1)/2$ c-number ones, $\xi^i_j = (\xi^j_i)^*$, for $i \neq j$. The advantage of this choice is that the parameters ξ^i_j can be directly associated to the c-number gauge fields due to the factor $1/\sqrt{2}$ which gives their correct normalization. In addition, this basis exhibit good trace orthogonality properties,

$$ Tr(D_{\hat{i}} D_{\hat{j}}) = \frac{1}{2} \delta_{\hat{i}\hat{j}}, \quad Tr(D_{\hat{i}} E^i_j) = 0, \quad Tr(E^i_j E^k_l) = \frac{1}{2} \delta^i_l \delta^k_j. \tag{1} $$

When we consider different irreps, ρ of the $su(n)$ algebra we denote $\xi^\rho = \rho(\xi)$ for each $\xi \in su(n)$ such that the corresponding basis-generators of the irrep ρ are $D^\rho_{\hat{i}} = \rho(D_{\hat{i}})$ and $E^{\rho\,i}_j = \rho(E^i_j)$.

2.2 Fermions

The $U(1)_Y$ transformations are nothing else but phase factor multiplications. Therefore - since the coupling constants g for $SU(n)_L$ and g' for the $U(1)_Y$ are assigned - the transformation of the fermion tensor L^ρ with respect to the gauge group $SU(n)_L \otimes$

$U(1)_Y$ of the theory reads

$$L^\rho \to U(\xi^0, \xi)L^\rho = e^{-i(g\xi^\rho + g'y_{ch}\xi^0)}L^\rho \tag{2}$$

where $\xi = \in su(n)$ and y_{ch} is the chiral hypercharge defining the irrep of the $U(1)_Y$ group parametrized by ξ^0. For simplicity, the general method deals with the character $y = y_{ch}g'/g$ instead of the chiral hypercharge y_{ch}, but this mathematical artifice does not affect in any way the results. Therefore, the irreps of the whole gauge group $SU(n)_L \otimes U(1)_Y$ are uniquely detemined by indicating the dimension of the $SU(n)$ tensor and its character y as $\rho = (\mathbf{n}_\rho, y_\rho)$.

In general, the spinor sector of our models has at least a part which is put in pure left form using the charge conjugation. Consequently this includes only left components, $L = \sum_\rho \oplus L^\rho$, that transform according to an arbitrary reducible representation of the gauge group. The Lagrangian density (Ld) of the free spinor sector has the form

$$\mathcal{L}_{S_0} = \frac{i}{2}\sum_\rho \overline{L^\rho}\overleftrightarrow{\partial\!\!\!/} L^\rho - \frac{1}{2}\sum_{\rho\rho'}\left(\overline{L^\rho}\chi^{\rho\rho'}(L^{\rho'})^c + h.c.\right). \tag{3}$$

Bearing in mind that each left-handed multiplet transforms as $L^\rho \to U^\rho(\xi^0, \xi)L^\rho$ we understand that $\mathcal{L}_{S_0}$ remains invariant under the global $SU(n)_L \otimes U(1)_Y$ transformations if the blocks $\chi^{\rho\rho'}$ transform like $\chi^{\rho\rho'} \to U^\rho(\xi^0, \xi)\chi^{\rho\rho'}(U^{\rho'}(\xi^0, \xi))^T$, according to the representations $(\mathbf{n}_\rho \otimes \mathbf{n}_{\rho'}, y_\rho + y_{\rho'})$ which generally are reducible. These blocks will give rise to the Yukawa terms.

2.3 Gauge fields

The spinor sector is coupled to the standard Yang-Mills sector constructed in usual manner by gauging the $SU(n)_L \otimes U(1)_Y$ symmetry. To this end we introduce the gauge fields $A^0_\mu = (A^0_\mu)^*$ and $A_\mu = A^+_\mu = A^a_\mu T_a \in su(n)$. Furthermore, the ordinary derivatives are replaced in Eq. (3) by the covariant ones, defined as $D_\mu L^\rho = \partial_\mu L^\rho - ig(A^\rho_\mu + y_\rho A^0_\mu)L^\rho$. Interaction terms occur

2.4 Minimal Higgs mechanism

The general method assumes also a particular Higgs mechanism (mHm) based on a special parametrization in the scalar sector, such that the n Higgs multiplets $\phi^{(1)}$, $\phi^{(2)}$, ... $\phi^{(n)}$ satisfy the orthogonality condition $\phi^{(i)+}\phi^{(j)} = \phi^2\delta_{ij}$ in order to eliminate the unwanted Goldstone bosons that could survive the SSB. ϕ is a gauge-invariant real scalar field while the Higgs multiplets $\phi^{(i)}$ transform according to the irreps $(\mathbf{n}, y^{(i)})$ whose characters $y^{(i)}$ are arbitrary numbers that can be organized into the diagonal matrix $Y = Diag\left(y^{(1)}, y^{(2)}, \cdots, y^{(n)}\right)$. The Higgs sector needs, in our approach, a parameter matrix

$$\eta = Diag\left(\eta^{(1)}, \eta^{(2)}, ..., \eta^{(n)}\right) \tag{4}$$

with the property $\text{Tr}(\eta^2) = 1 - \eta_0^2$. It will play the role of the metric in the kinetic part of the Higgs Ld which reads

$$\mathcal{L}_H = \frac{1}{2}\eta_0^2 \partial_\mu \phi \partial^\mu \phi + \frac{1}{2}\sum_{i=1}^{n} \left(\eta^{(i)}\right)^2 \left(D_\mu \phi^{(i)}\right)^+ \left(D^\mu \phi^{(i)}\right) - V(\phi) \tag{5}$$

where $D_\mu \phi^{(i)} = \partial_\mu \phi^{(i)} - ig(A_\mu + y^{(i)} A_\mu^0)\phi^{(i)}$ are the covariant derivatives of the model and $V(\phi)$ is the scalar potential generating the SSB of the gauge symmetry [5]. This is assumed to have an absolute minimum for $\phi = \langle\phi\rangle \neq 0$ that is, $\phi = \langle\phi\rangle + \sigma$ where σ is the unique surviving physical Higgs field. Therefore, one can always define the unitary gauge where the Higgs multiplets, $\hat{\phi}^{(i)}$ have the components $\hat{\phi}_k^{(i)} = \delta_{ik}\phi = \delta_{ik}(\langle\phi\rangle + \sigma)$.

2.5 Physical bosons

The next step is to find the physical neutral bosons. Therefore, fist one has to separate the electromagnetic potential A_μ^{em} corresponding to the surviving $U(1)_{em}$ symmetry. The one-dimensional subspace of the parameters ξ^{em} associated to this symmetry assumes a particular direction in the parameter space $\{\xi^0, \xi^{\hat{i}}\}$ of the whole Cartan sub-algebra. This is uniquely determined by the $n - 1$ - dimensional unit vector ν and the angle θ giving the subspace equations $\xi^0 = \xi^{em}\cos\theta$ and $\xi^{\hat{i}} = \nu_{\hat{i}}\xi^{em}\sin\theta$. On the other hand, since the Higgs multiplets in unitary gauge remain invariant under $U(1)_{em}$ transformations, we must impose the obvious condition $D_{\hat{i}}\xi^{\hat{i}} + Y\xi^0 = 0$ which yields $Y = -D_{\hat{i}}\nu^{\hat{i}}\tan\theta \equiv -(D \cdot \nu)\tan\theta$.

In other words, the new parameters (ν, θ) determine all the characters $y^{(i)}$ of the irreps of the Higgs multiplets. For this reason these will be considered the principal parameters of the model and therefore one deals with θ and ν (which has $n - 2$ independent components) instead of $n - 1$ parameters $y^{(i)}$.

Under these circumstances, the generating mass term

$$\frac{g^2}{2}\langle\phi\rangle^2 Tr\left[\left(A_\mu + YA_\mu^0\right)\eta^2\left(A^\mu + YA^{0\mu}\right)\right] , \tag{6}$$

depends now on the parameters θ and $\nu_{\hat{i}}$. The neutral bosons in Eq. 6 being the electromagnetic field A_μ^{em} and the $n - 1$ new ones, $A_\mu^{'\hat{i}}$, which are the diagonal bosons remaining after the separation of the electromagnetic potential [5].

This term straightforwardly gives rise to the masses of the non-diagonal gauge bosons

$$M_i^j = \frac{1}{2}g\langle\phi\rangle \sqrt{\left[\left(\eta^{(i)}\right)^2 + \left(\eta^{(j)}\right)^2\right]}, \tag{7}$$

while the masses of the neutral bosons $A_\mu^{'\hat{i}}$ have to be calculated by diagonalizing the matrix

$$(M^2)_{\hat{i}\hat{j}} = \langle\phi\rangle^2 Tr(B_{\hat{i}}B_{\hat{j}}) \tag{8}$$

where

$$B_{\hat{i}} = g\left(D_{\hat{i}} + \nu_{\hat{i}}(D \cdot \nu)\frac{1 - \cos\theta}{\cos\theta}\right)\eta, \tag{9}$$

4

As it was expected, A_μ^{em} does not appear in the mass term and, consequently, it remains massless. The other neutral gauge fields $A'^{\hat{i}}_\mu$ have the non-diagonal mass matrix (8). This can be brought in diagonal form with the help of a new $SO(n-1)$ transformation, $A'^{\hat{i}}_\mu = \omega^{\hat{i}}_{.\hat{j}} Z^{\hat{j}}_\mu$, which leads to the physical neutral bosons $Z^{\hat{i}}_\mu$ with well-defined masses. Performing this $SO(n-1)$ transformation the physical neutral bosons are completely determined. The transformation

$$
\begin{aligned}
A_\mu^0 &= A_\mu^{em} \cos\theta - \nu_{\hat{i}} \omega^{\hat{i}}_{.\hat{j}} Z^{\hat{j}}_\mu \sin\theta, &(10)\\
A_\mu^{\hat{k}} &= \nu^{\hat{k}} A_\mu^{em} \sin\theta + \left(\delta^{\hat{k}}_{\hat{i}} - \nu^{\hat{k}}\nu_{\hat{i}}(1-\cos\theta)\right) \omega^{\hat{i}}_{.\hat{j}} Z^{\hat{j}}_\mu. &(11)
\end{aligned}
$$

which switches from the original diagonal gauge fields, $(A_\mu^0, A_\mu^{\hat{i}})$ to the physical ones, $(A_\mu^{em}, Z^{\hat{i}}_\mu)$ is called the generalized Weinberg transformation (gWt).

2.6 Electric and Neutral Charges

The nest step is to identify the charges of the particles with the coupling coefficients of the currents with respect to the above determined physical bosons. Thus, we find that the spinor multiplet L^ρ (of the irrep ρ) has the following electric charge matrix

$$
Q^\rho = g\left[(D^\rho \cdot \nu)\sin\theta + y_\rho \cos\theta\right], \tag{12}
$$

and the $n-1$ neutral charge matrices

$$
Q^\rho(Z^{\hat{i}}) = g\left[D^\rho_{\hat{k}} - \nu_{\hat{k}}(D^\rho \cdot \nu)(1-\cos\theta) - y_\rho \nu_{\hat{k}} \sin\theta\right] \omega^{\hat{k}}_{.\hat{i}} \tag{13}
$$

corresponding to the $n-1$ neutral physical fields, $Z^{\hat{i}}_\mu$. All the other gauge fields, namely the charged bosons $A^i_{j\mu}$, have the same coupling, $g/\sqrt{2}$, to the fermion multiplets.

3 $SU(4)_L \otimes U(1)_Y$ models without exotic charges

The general method - constructed in Ref. [5] and briefly presented in the above section - is based on the following assumptions in order to give viable results when it is applied to concrete models:

(I) the spinor sector must be put (at least partially) in pure left form using the charge conjugation (see for details Appendix B in Ref. [5])

(II) the minimal Higgs mechanism - with arbitrary parameters (η_0, η) satisfying the condition $\mathrm{Tr}(\eta^2) = 1 - \eta_0^2$ and giving rise to traditional Yukawa couplings in unitary gauge - must be employed

(III) the coupling constant, g, is the same with the first one of the SM

(IV) at least one Z-like boson should satisfy the mass condition $m_Z = m_W/\cos\theta_W$ established in the SM and experimentally confirmed.

Bearing in mind all these necessary ingredients, we proceed to solving the particular 3-4-1 model [3] by imposing from the very beginning the set of parameters

we will work with. In the following, we will use the standard generators T_a of the $su(4)$ algebra. Therefore, as the Hermitian diagonal generators of the Cartan subalgebra one deals, in order, with $D_1 = T_3 = \frac{1}{2}Diag(1, -1, 0, 0)$, $D_2 = T_8 = \frac{1}{2\sqrt{3}}Diag(1, 1, -2, 0)$, and $D_3 = T_{15} = \frac{1}{2\sqrt{6}}Diag(1, 1, 1, -3)$ respectively. At the same time, we denote the irreps of the electroweak model under consideration here by $\rho = (\mathbf{n}_\rho, y_{ch}^\rho)$ indicating the genuine chiral hypercharge y_{ch} instead of y. Therefore, the multiplets - subject to anomaly cancellation - of the 3-4-1 model of interest here will be denoted by $(\mathbf{n}_{color}, \mathbf{n}_\rho, y_{ch}^\rho)$.

There are three distinct cases [6] leading to a discrimination among models of the 3-4-1 class, according to their electric charge assignment. They are: (i) versors $\nu_1 = 1$, $\nu_2 = 0$, $\nu_3 = 0$, (ii) versors $\nu_1 = 0$, $\nu_2 = 1$, $\nu_3 = 0$, and (iii) versors $\nu_1 = 0$, $\nu_2 = 0$, $\nu_3 = -1$, respectively. At the same time, one assumes the condition $e = g \sin\theta_W$ established in the SM.

3.1 Fermion content

With this notation, after little algebra involving Eqs. (12) - (14) and the versor setting $\nu_1 = 0$, $\nu_2 = 0$, $\nu_3 = -1$ - Case 3 in ref. [6] - one finds two distinct classes of 3-4-1 models without exotic electric charges:First of all, let's observe that no 4-plet obeys the fundamental irrep of the gauge group $\rho = (\mathbf{4}, 0)$. Notwithstanding, since for the lepton 4-plet one can assign two different chiral hypercharges $-\frac{1}{4}$ and $-\frac{3}{4}$ respectively, we get two sub-cases leading to two different versions of 3-4-1 anomaly-free models without exotic electric charges. The coupling matching, as we will see in the following, assumes the same relation in both sub-cases.

From Eq. (12), it is straightforward that the lepton family exhibits the electric charge operator

$$Q^{(4^*, -\frac{1}{4})} = e\left[-T_{15}^{(4^*)}\frac{\sin\theta}{\sin\theta_W} - \frac{1}{4}\left(\frac{g'}{g}\right)\frac{\cos\theta}{\sin\theta_W}\right], \tag{14}$$

for the first choice. This leads to the lepton representation $\left(N'_\alpha, \ N_\alpha, \ \nu_\alpha, \ e_\alpha\right)_L^T \sim (\mathbf{4}^*, -\frac{1}{4})$ including two new kinds of neutral leptons (N_α, N'_α). For the second choice, the electric charge operator will be represented as

$$Q^{(4, -\frac{1}{4})} = e\left[-T_{15}^{(4)}\frac{\sin\theta}{\sin\theta_W} - \frac{3}{4}\left(\frac{g'}{g}\right)\frac{\cos\theta}{\sin\theta_W}\right], \tag{15}$$

leading to the lepton families $\left(E'^-_\alpha, \ E^-_\alpha, \ e^-_\alpha, \ \nu_\alpha\right)_L^T \sim (\mathbf{4}, -\frac{3}{4})$ that allow for new charged leptons $(E^-_\alpha, E'^-_\alpha)$.

After a little algebra, both Eqs (14) and (15) require - via the compulsory condition $\sin\theta = \sqrt{\frac{3}{2}}\sin\theta_W$, since the only allowed electric charges in the lepton sector are 0 and $\pm e$ - the coupling matching: $\frac{g'}{g} = \frac{\sin\theta_W}{\sqrt{1-\frac{3}{2}\sin^2\theta_W}}$.

Once these assingments are assumed, the quarks will aquire their electric charges from the following operators

$$Q^{(4^*,\frac{5}{12})} = e\left[-T_{15}^{(4^*)}\frac{\sin\theta}{\sin\theta_W} + \frac{5}{12}\left(\frac{g'}{g}\right)\frac{\cos\theta}{\sin\theta_W}\right] \tag{16}$$

$$Q^{(4,-\frac{1}{12})} = e\left[-T_{15}^{(4)}\frac{\sin\theta}{\sin\theta_W} - \frac{1}{12}\left(\frac{g'}{g}\right)\frac{\cos\theta}{\sin\theta_W}\right] \tag{17}$$

3.1.1 Model A

With the first of the above mentioned assumptions, the fermion representations are:

Lepton families

$$f_{\alpha L} = \begin{pmatrix} N'_\alpha \\ N_\alpha \\ \nu_\alpha \\ e_\alpha \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{4}^*, -1/4) \qquad (e_{\alpha L})^c \sim (\mathbf{1}, \mathbf{1}, 1) \tag{18}$$

Quark families

$$Q_{iL} = \begin{pmatrix} D'_i \\ D_i \\ -d_i \\ u_i \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{4}, -1/12) \qquad Q_{3L} = \begin{pmatrix} U' \\ U \\ u_3 \\ d_3 \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{4}^*, 5/12)$$
$$\tag{19}$$
$$(d_{3L})^c, (d_{iL})^c, (D_{iL})^c, (D'_{iL})^c \sim (\mathbf{3}, \mathbf{1}, +1/3) \tag{20}$$

$$(u_{3L})^c, (u_{iL})^c, (U_L)^c, (U'_L)^c \sim (\mathbf{3}, \mathbf{1}, -2/3) \tag{21}$$

with $\alpha = 1, 2, 3$ and $i = 1, 2$. We recovered the same fermion content as the one of the model presented in Refs. [3, 4].

3.1.2 Model B

With the second of the above mentioned assumptions, the fermion representations are:

Lepton families

$$f_{\alpha L} = \begin{pmatrix} E'^-_\alpha \\ E^-_\alpha \\ e^-_\alpha \\ e_\alpha \end{pmatrix}_L \sim (\mathbf{1}, \mathbf{4}, -3/4) \qquad (e_{\alpha L})^c, (E_{\alpha L})^c, (E'_{\alpha L})^c \sim (\mathbf{1}, \mathbf{1}, 1)$$
$$\tag{22}$$

Quark families

$$Q_{iL} = \begin{pmatrix} U'_i \\ U_i \\ u_i \\ d_i \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{4}^*, 5/12) \qquad Q_{3L} = \begin{pmatrix} D' \\ D \\ -d_3 \\ u_3 \end{pmatrix}_L \sim (\mathbf{3}, \mathbf{4}, -1/12)$$
$$\tag{23}$$

$$(d_{3L})^c, (d_{iL})^c, (D_{iL})^c, (D'_{iL})^c \sim (\mathbf{3}, \mathbf{1}, +1/3) \tag{24}$$

$$(u_{3L})^c, (u_{iL})^c, (U_L)^c, (U'_L)^c \sim (\mathbf{3}, \mathbf{1}, -2/3) \tag{25}$$

with $\alpha = 1, 2, 3$ and $i = 1, 2$. We recovered the same fermion content as the one of the model presented in Refs. [3, 4].

With this assignment the fermion families (in each of the above displayed cases) cancel the axial anomalies by just an interplay between them, although each family remains anomalous by itself. Thus, the renormalization criteria are fulfilled and the method is validated once more from this point of view. Note that one can add at any time sterile neutrinos - *i.e.* right-handed neutrinos $\nu_{\alpha R} \sim (\mathbf{1}, \mathbf{1}, 0)$ - that could pair in the neutrino sector of the Ld with left-handed ones in order to eventually generate tiny Dirac or Majorana masses by means of an adequate see-saw mechanism. These sterile neutrinos do not affect anyhow the anomaly cancelation, since all their charges are zero. Moreover, their number is not restricted by the number of flavors in the model

3.2   Boson mass spectrum

Subsequently, we will use the standard generators T_a of the $su(4)$ algebra. In this basis, the gauge fields are A_μ^0 and $A_\mu \in su(4)$, that is

$$A_\mu = \frac{1}{2} \begin{pmatrix} D_\mu^1 & \sqrt{2}Y_\mu & \sqrt{2}X'_\mu & \sqrt{2}X'_\mu \\ \sqrt{2}Y_\mu^* & D_\mu^2 & \sqrt{2}K_\mu & \sqrt{2}K'_\mu \\ \sqrt{2}X_\mu^* & \sqrt{2}K_\mu^* & D_\mu^3 & \sqrt{2}W_\mu \\ \sqrt{2}X'^*_\mu & \sqrt{2}K'^*_\mu & \sqrt{2}W_\mu^* & D_\mu^4 \end{pmatrix}, \tag{26}$$

with $D_\mu^1 = A_\mu^3 + A_\mu^8/\sqrt{3} + A_\mu^{15}/\sqrt{6}$, $D_\mu^2 = -A_\mu^3 + A_\mu^8/\sqrt{3} + A_\mu^{15}/\sqrt{6}$, $D_\mu^3 = -2A_\mu^8/\sqrt{3} + A_\mu^{15}/\sqrt{6}$, $D_\mu^4 = -3A_\mu^{15}/\sqrt{6}$ as diagonal bosons. Apart from the charged Weinberg bosons ($W^\pm$), there are two new charged bosons, K^0, $K'^\pm$, while X^0, $X'^\pm$ and Y^0 are new neutral bosons, but distinct from the diagonl ones.

The masses of both the neutral and charged bosons depend on the choice of the matrix η whose components are free parameters. Here it is convenient to assume the following matrix

$$\eta^2 = (1 - \eta_0^2)Diag\left(1 - c, c - a, \frac{1}{2}a + b, \frac{1}{2}a - b\right), \tag{27}$$

where, for the moment, a, b and c are arbitrary non-vanishing real parameters. Obviously, $\eta_0, c \in [0, 1)$, $a \in (0, c)$ and $b \in (-a, +a)$. Note that with this parameter choice the condition (II) is accomplished.

Under these circumstances, the mass spectrum corresponding to the off-diagonal bosons (usually, charged ones), according to Eq. (7) reads

$$m^2(W) = m^2 a, \tag{28}$$

$$m^2(X) = m^2\left(1 - c + \frac{1}{2}a + b\right), \tag{29}$$

$$m^2(X') = m^2\left(1 - c + \frac{1}{2}a - b\right), \tag{30}$$

$$m^2(K) = m^2\left(c - \frac{1}{2}a + b\right), \tag{31}$$

$$m^2(K') = m^2\left(c - \frac{1}{2}a - b\right), \tag{32}$$

$$m^2(Y) = m^2\left(1 - a\right). \tag{33}$$

while the mass matrix of the neutral bosons is given by Eq. (8)

$$M^2 = m^2 \begin{pmatrix} (1-a) & \frac{1-2c+a}{\sqrt{3}} & \frac{1-2c+a}{\sqrt{6}\cos\theta} \\ \frac{1-2c+a}{\sqrt{3}} & \frac{1}{3}(1+a+4b) & \frac{1-2a-2b}{3\sqrt{2}\cos\theta} \\ \frac{1-2c+a}{\sqrt{6}\cos\theta} & \frac{1-2a-2b}{3\sqrt{2}\cos\theta} & \frac{1+4a-8b}{6\cos^2\theta} \end{pmatrix} \tag{34}$$

with $m^2 = g^2 \langle\phi\rangle^2 (1 - \eta_0^2)/4$ throughout this paper. In order to fulfil the requirement (IV), the above matrix has to admit $m^2 a/\cos^2 \theta_W$ as eigenvalue, that is one has to compute $Det\left|M^2 - \frac{m^2 a}{\cos^2 \theta_W}\right| = 0$. Now, one can enforce some other phenomenological assumptions. First of all, it is natural to presume that the third neutral (diagonal boson) Z'' should be considered much heavier than its companions, so that it decouples form their mixing, as the symmetry is broken to $SU(3)$. For this purpose a higher breaking scale is responsable.

Therefore $M_{12}^2 = M_{21}^2 = M_{13}^2 = M_{31}^2$ in the matrix (34). This gives rise to the natural condition

$$c = \frac{1+a}{2}, \tag{35}$$

in order to vanish the above terms.

Hence, Eq. (34) looks like

$$M^2 = m^2 \begin{pmatrix} (1-a) & 0 & 0 \\ 0 & \frac{1+a+4b}{3} & \frac{1-2a-2b}{3\sqrt{2}\cos\theta} \\ 0 & \frac{1-2a-2b}{3\sqrt{2}\cos\theta} & \frac{1+4a-8b}{6\cos^2\theta} \end{pmatrix} \tag{36}$$

9

Let us observe that the condition (IV) - via computing $Det \left| M^2 - \frac{m^2 a}{\cos^2 \theta_W} \right| = 0$ - is fulfilled if and only if $b = \frac{1}{2} a \tan^2 \theta_W$, resulting from diagonalization of the remaning part of the matrix (36). Therefore, one finally remains with only one parameter - say a.

Obviously, Z is the neutral boson of the SM, while Z' is a new neutral boson of this model (also occuring in 3-3-1 models) whose mass comes form $Tr(M^2) = m^2(Z) + m^2(Z') + m^2(Z'')$.

With these preliminaries, the boson mass spectrum holds:

$$m^2(W) = m^2 a, \tag{37}$$

$$m^2(X) = m^2 a \left(\frac{1 + \tan^2 \theta_W}{2} \right), \tag{38}$$

$$m^2(X') = m^2 a \left(\frac{1 - \tan^2 \theta_W}{2} \right), \tag{39}$$

$$m^2(K) = m^2 a \left(\frac{1 + \tan^2 \theta_W}{2} \right), \tag{40}$$

$$m^2(K') = m^2 a \left(\frac{1 - \tan^2 \theta_W}{2} \right), \tag{41}$$

$$m^2(Y) = m^2(1 - a), \tag{42}$$

$$m^2(Z) = m^2 a / \cos^2 \theta_W, \tag{43}$$

$$m^2(Z') = m^2 \frac{\cos^4 \theta_W - a \sin^4 \theta_W}{\cos^2 \theta_W \left(2 - 3 \sin^2 \theta_W \right)}, \tag{44}$$

$$m^2(Z'') = m^2(1 - a). \tag{45}$$

The mass scale is now just a matter of tuning the parameter a in accordance with the possible values for $\langle \phi \rangle$.

3.3 Neutral charges

Now one can compute in detail all the charges for the fermion representations in models A and B with respect to the neutral bosons (Z, Z', Z''), since the gWt is determined by the matrix

$$\omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}\sqrt{1 - \sin^2 \theta_W}} & \frac{\sqrt{2 - 3\sin^2 \theta_W}}{\sqrt{3}\sqrt{1 - \sin^2 \theta_W}} \\ 0 & -\frac{\sqrt{2 - 3\sin^2 \theta_W}}{\sqrt{3}\sqrt{1 - \sin^2 \theta_W}} & \frac{1}{\sqrt{3}\sqrt{1 - \sin^2 \theta_W}} \end{pmatrix}. \tag{46}$$

These will be expressed - via Eq.(13) with the versor assignment $\nu_1 = 0$, $\nu_2 = 0$, $\nu_3 = -1$ - by:

$$Q^\rho(Z^{\hat{i}}) = g\left[D_1^\rho \omega^1_{\cdot \hat{i}} + D_2^\rho \omega^2_{\cdot \hat{i}} + \left(D_3^\rho \cos\theta + y_{ch}^\rho \frac{g'}{g}\sin\theta\right)\omega^3_{\cdot \hat{i}}\right], \qquad (47)$$

where the conditions $\frac{g'}{g} = \frac{\sin\theta_W}{\sqrt{1-\frac{3}{2}\sin^2\theta_W}}$ and $\sin\theta = \sqrt{\frac{3}{2}}\sin\theta_W$ have to be inserted.

Evidently, the heaviest neutral boson - $Z^1 = Z''$, in our notation - will couple the fermion representations through:

$$Q^\rho(Z^1) = gD_1^\rho \qquad (48)$$

while the other two - $Z^2 = Z$ of the SM, and $Z^3 = Z'$ respectively - exhibit the following charges:

$$Q^\rho(Z^2) = g\left[D_2^\rho \omega^2_{\cdot 2} + \left(D_3^\rho \sqrt{1-\frac{3}{2}\sin^2\theta_W} + y_{ch}^\rho \frac{\sqrt{3}\sin^2\theta_W}{\sqrt{2-3\sin^2\theta_W}}\right)\omega^3_{\cdot 2}\right] \qquad (49)$$

$$Q^\rho(Z^3) = g\left[D_2^\rho \omega^2_{\cdot 3} + \left(D_3^\rho \sqrt{1-\frac{3}{2}\sin^2\theta_W} + y_{ch}^\rho \frac{\sqrt{3}\sin^2\theta_W}{\sqrt{2-3\sin^2\theta_W}}\right)\omega^3_{\cdot 3}\right] \qquad (50)$$

Assuming the ω- matrix given by Eq. (46), the neutral charges of the fermions in the two models under consideration here are computed and listed in Tables 1 and 2.

4   Conclusions

Regarding the neutral currents, one can observe that the leptons and quarks of the SM recover their known values with respect to the Z boson, while each exotic fermion in the 3-4-1 models of interest here exhibit a vector coupling with respect to the same boson, *i.e.* its left-handed component and its right-handed one are indistinct in interaction with Z. On the other hand, Z'' couples only the exotic fermions.

In order to allow for a high breaking scale in the model $\langle\phi\rangle \geq 1\text{TeV}$ and keep at the same time consistency with low energy phenomenology of the SM our solution favors the case with $a \to 0$ and $c \to \frac{1}{2}$. However, assuming that $m(W) \simeq 84.4\text{GeV}$ and $m(Z) \simeq 91.2\text{GeV}$ and $\sin^2\theta_W \simeq 0.223$ [9], our approach predicts the exact masses at tree level for the following bosons (according to Eqs. (37) - (45)): $m(X) \equiv m(K) \simeq 67.7\text{GeV}$, $m(X') \equiv m(K') \simeq 50.4\text{GeV}$ which are independent of the precise account of the overall vev of the model. The heavier bosons - essentially depending on the precise account of the vev, and hence on the parameter a- fulfil the following hierarchy $m(Z'') \equiv m(Y) > m(Z') > m(Z) > m(W) > m(X) \equiv m(K) > m(X') \equiv m(K')$.

We can offer here a rough estimate. If the mass scale of the model (m in our notation), lies in the TeV region or in a higher one, then from Eqs. 37 , with $m \simeq 1\text{TeV}$, $a \simeq 0.007$ is inferred. Hence, by working out Eqs. (42) (44) and (45) the rest of the bosons are given their masses accordingly: $m(Z'') \equiv m(Y) \simeq 0.996\text{TeV}$ and

Table 1: Coupling coefficients of the neutral currents in 3-4-1 model A

Particle\Coupling($e/\sin 2\theta_W$)	$Z \to ff$	$Z' \to ff$	$Z'' \to ff$
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	1	$\dfrac{1-3\sin^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
e_L, μ_L, τ_L	$2\sin^2\theta_W - 1$	$\dfrac{1-3\sin^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
$N_{eL}, N_{\mu L}, N_{\tau L}$	0	$-\dfrac{3\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$
$N'_{eL}, N'_{\mu L}, N'_{\tau L}$	0	$-\dfrac{3\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$
e_R, μ_R, τ_R	$2\sin^2\theta_W$	$-\dfrac{2\sin^2\theta_W}{\sqrt{2-3\sin^2\theta_W}}$	0
u_L, c_L	$1 - \frac{4}{3}\sin^2\theta_W$	$\dfrac{2-9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
d_L, s_L	$-1 + \frac{2}{3}\sin^2\theta_W$	$\dfrac{2-9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
t_L	$1 - \frac{4}{3}\sin^2\theta_W$	$\dfrac{2+9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	0
b_L	$-1 + \frac{2}{3}\sin^2\theta_W$	$\dfrac{2+9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	0
$u_R, c_R, t_R, U_{1R}, U'_{iR}$	$-\frac{4}{3}\sin^2\theta_W$	$\dfrac{4\sin^2\theta_W}{3\sqrt{2-3\sin^2\theta_W}}$	0
$d_R, s_R, b_R, D_{iR}, D'_{iR}$	$+\frac{2}{3}\sin^2\theta_W$	$-\dfrac{2\sin^2\theta_W}{3\sqrt{2-3\sin^2\theta_W}}$	0
D_{1L}, D_{2L}	$\frac{2}{3}\sin^2\theta_W$	$\dfrac{5-9\sin^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$
D'_{1L}, D'_{2L}	$\frac{2}{3}\sin^2\theta_W$	$\dfrac{5-9\sin^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$
U_{3L}	$-\frac{4}{3}\sin^2\theta_W$	$\dfrac{-1+9\sin^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$
U'_{3L}	$-\frac{4}{3}\sin^2\theta_W$	$\dfrac{-1+9\sin^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$

Table 2: Coupling coefficients of the neutral currents in 3-4-1 Model B

Particle\Coupling($e/\sin 2\theta_W$)	$Z \to ff$	$Z' \to ff$	$Z'' \to ff$
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	1	$\dfrac{-5+3\sin^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
e_L, μ_L, τ_L	$2\sin^2\theta_W - 1$	$\dfrac{-5+3\sin^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
$E_{eL}, E_{\mu L}, E_{\tau L}$	$2\sin^2\theta_W$	$-\dfrac{1+3\sin^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$
$E'_{eL}, E'_{\mu L}, E'_{\tau L}$	$2\sin^2\theta_W$	$-\dfrac{1+3\sin^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$
e_{iR}, E_{iR}, E'_{iR}	$2\sin^2\theta_W$	$-\dfrac{2\sin^2\theta_W}{\sqrt{2-3\sin^2\theta_W}}$	0
u_L, c_L	$1 - \frac{4}{3}\sin^2\theta_W$	$\dfrac{2+9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	0
d_L, s_L	$-1 + \frac{2}{3}\sin^2\theta_W$	$\dfrac{2+9\cos^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	0
t_L	$1 - \frac{4}{3}\sin^2\theta_W$	$\dfrac{2-9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
b_L	$-1 + \frac{2}{3}\sin^2\theta_W$	$\dfrac{2-9\cos^2\theta_W}{2\sqrt{2-3\sin^2\theta_W}}$	0
$u_R, c_R, t_R, U_{1R}, U'_{iR}$	$-\frac{4}{3}\sin^2\theta_W$	$\dfrac{4\sin^2\theta_W}{3\sqrt{2-3\sin^2\theta_W}}$	0
$d_R, s_R, b_R, D_{iR}, D'_{iR}$	$+\frac{2}{3}\sin^2\theta_W$	$-\dfrac{2\sin^2\theta_W}{3\sqrt{2-3\sin^2\theta_W}}$	0
D_{3L}	$\frac{2}{3}\sin^2\theta_W$	$\dfrac{5-9\sin^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$
D'_{3L}	$\frac{2}{3}\sin^2\theta_W$	$\dfrac{5-9\sin^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$
U_{1L}, U_{2L}	$-\frac{4}{3}\sin^2\theta_W$	$\dfrac{-1+9\sin^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$\cos\theta_W$
U'_{1L}, U'_{2L}	$-\frac{4}{3}\sin^2\theta_W$	$\dfrac{-1+9\sin^2\theta_W}{6\sqrt{2-3\sin^2\theta_W}}$	$-\cos\theta_W$

$m(Z') \simeq 0.55\text{TeV}$. A more accurate estimate for the masses of these bosons and the relations among them (by a more apropriate tuning of parameter a) can be done, once the experimental evidence of their phenomenology will be definitely bring to light at LHC, LEP, CDF, Tevatron and other high energy accelerators in a near future.

References

[1] S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967), S. L. Glashow, *Nucl. Phys.* **22**, 579 (1961), A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No.8), ed. N. Svartholm (Almqvist and Wiksell, 1968) p.367.

[2] R. Foot, H. N. Long and T. A. Tran, *Phys. Rev. D* **50**, R34 (1994); F. Pisano and V. Pleitez, *Phys. Rev. D* **51**, 3865 (1995); A. Doff and F. Pisano, *Mod. Phys. Lett. A* **14**, 1133 (1999); A. Doff and F. Pisano, *Phys. Rev. D* **63**, 097903 (2001); Fayyazuddin and Riazuddin, *JHEP* **0412**, 013 (2004), A. Palcu, arXiv: 0902.1828 (hep-ph).

[3] W. A. Ponce, D. A. Gutierrez and L. A. Sanchez, *Phys. Rev. D* **69**, 055007 (2004); L. A. Sanchez, L. A. Wills-Toro and J. I. Zuluaga, *Phys. Rev. D* **77**, 035008 (2008); Riazuddin and Fayyazuddin, *Eur. Phys. J C* **56**, (2008).

[4] A. Doff and F. Pisano, *Mod. Phys. Lett. A* **15**, 1471 (2000); W. A. Ponce and L. A. Sanchez, *Mod. Phys. Lett. A* **22**, 435 (2007).

[5] I. I. Cotăescu, *Int. J. Mod. Phys. A* **12**, 1483 (1997).

[6] A. Palcu, arXiv: 0902.1301 (hep-ph).

[7] L. A. Sanchez, F. A Perez and W. A. Ponce, *Eur. Phys. J C* **35**, 259 (2004).

[8] R. Gilmore, Lie Groups, Lie Algebras and some of their Applications (Wiley Interscience, 1974); A. O. Barut and R. Raczka, Theory of Group Representations and Applications (PWN, 1977).

[9] Particle Data Group (C. Amsler *et al.*), *Phys. Lett. B* **667**, 1 (2008).